solve trinomial equations of all degrees with any coefficients. S. Gundelfinger, *Tafeln zur Berechnung der reellen Wurzeln sämtlicher trinomischer Gleichungen* (Leipzig, 1897).

For references to the literature of trinomial equations, see the article, R. Mehmke, *Calculs numériques*, in the *Encyclopédie des Sciences Mathématiques*, tome I, Volume 4, I 23.

In § 41, pages 320-325, the following method is given.

In $A\beta^2+C\beta^{1.6}=W$, put $\beta=kx^n$. Then $Ak^2x^{2n}+Ck^{1.6}x^{1.6n}=W$.

Let $1.6n=1$, whence $n=\frac{1}{1.6}=.625$.

Let $Ak^2=Ck^{1.6}$, whence $Ak^{0.4}=C$, and $k=\left(\frac{C}{A}\right)^{2.5}$.

Then equation reduces to $x^{1.25}+x=D$.

Make a table of values of $x^{1.25}+x$ for different values of x with as small an interval as desired. Then any given equation may be solved by simple interpolation.

For graphic solution (by nomography), see Articles 45, 51, 61; also M. d'Ocagne, *Traité de Nomographie* (Paris, 1899), pages 367, 387.

Another method is given by F. Schlepps, *Euber die Auflösung trinomischer Gleichungen aller Grade* (Halle a. S. (1899), 15 pages). [See *Jahrbuch ü. d. Fortschritté der Math.*, Vol. 30, page 104.]

GEOMETRY.

400. Proposed by FRANCIS RUST, C. E., Pittsburgh, Pennsylvania.

Given a circle and a point P without; construct, *using the straight edge only*, the two tangents to the circle through P.

Solution by H. E. TREFETHEN, Colby College, Waterville, Maine; H. C. FEEMSTER, York College, York, Nebraska, and ELMER SCHUYLER, New York City.

From P draw two secants and complete the inscribed quadrilateral thus determined. Let PQ be the external diagonal and R the point of intersection of the internal diagonals. Hence each side of the triangle PQR is the polar of the opposite vertex and QR cuts the circle in S and T, the points of contact of the required tangents PS and PT. Thus the construction is effected with the ruler only.

Also solved by M. E. Graber and E. B. Escott.

401. Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Find by Euclidean geometry a point whose distances from the vertices of an equilateral triangle are in the ratio 3:4:5. The general case of ratio $a{:}b{:}c$ would prove interesting.

I. Solution by W. J. GREENSTREET, M. A., Editor The Mathematical Gazette, Burgfield, England.

Let ABC be any triangle. The locus of a point P such that $PA{:}PB=x{:}y$ is well known to be a circle. Let the circular loci of the points P given

by $PA:PB=x:y$, $PB:PC=y:z$ cut in P. Then since $PC:PA=z:x$, it follows that P lies on the third of three circular loci given by $PA:PB:PC=x:y:z$, and these are three coaxial circles.

Each is orthogonal to the circle ABC. Hence the two points of intersection of the three loci are inverse points with respect to the circle ABC.

Hence two points satisfying the conditions can be found.

It follows that when P is on the circle ABC the two solutions reduce to one. Apply Ptolemy's theorem and we have

$$a'x+b'y=c'z, \quad b'y+c'z=a'x, \quad \text{or} \quad c'z+a'x=b'y,$$

where a', b', c' are proportional to the sides of the triangle ABC. Hence, if ABC is equilateral, we have

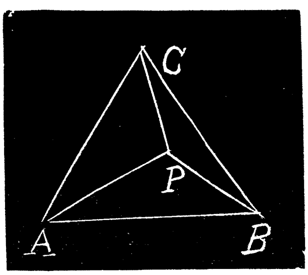
$$x+y=z, \quad y+z=x, \quad \text{or} \quad z+x=y,$$

i. e., one of the quantities x, y, z is equal to the sum of the other two.

This gives a solution of the problem. Therefore, construct a triangle of given species with its vertices on three concentric circles, radii x, y, z.

II. **Solution by J. SCHEFFRR, A. M., Hagerstown, Maryland.**

The following construction is valid for any triangle. Divide AB harmonically in the ratio $a:b$. On the two conjugate points construct a circle. Divide AC harmonically in the ratio $a:c$, and construct a circle on the line connecting the two conjugate points. The points common to the two circles satisfy the condition; for it is a well known theorem in geometry, that the circle erected on the distance of the two harmonic points of a straight line is the locus of the point whose distances from the extremities of the line are in the ratio in which the line is divided harmonically. There are consequently in general two points. The limitation is that the two circles either intersect or touch each other.



Treating the problem algebraically, let $AB=AC=BC=m$, $AP=ax$, $BP=bx$, $CP=cx$. Denoting $\angle PAB$ by θ, we have

$$\cos\theta=\frac{a^2x^2+m^2-b^2x^2}{2amx}, \quad \cos(60^\circ-\theta)=\frac{a^2x^2+m^2-c^2x^2}{2amx};$$

$$\therefore \tfrac{1}{2}\frac{a^2x^2+m^2-c^2x^2}{2amx}+\tfrac{1}{2}\sqrt{3}\sqrt{1-\frac{(a^2x^2+m^2-b^2x^2)^2}{4a^2m^2x^2}}=\frac{a^2x^2+m^2-c^2x^2}{2amx}.$$

Removing the radical and making all necessary reductions we finally get

$$(a^4+b^4+c^4-a^2b^2-a^2c^2-b^2c^2)x^4-(a^2+b^2+c^2)m^2x^2=-m^4.$$

Also solved by M. A. Harding, H. Prime, C. N. Schmall, and H. C. Feemster.

NOTES AND NEWS.

About 130 pages of the part of the French mathematical encyclopedia which was issued in June, 1912, are devoted to contemporary researches on the theory of functions. The three main subjects treated are the theory of concrete sets of points, the theories of integration and of finding derivatives, and the development into series. As regards the theory of sets of points, it is observed on page 115 that, in a very general way, it might be said that the German and English writers devote most attention to the abstract theory of sets of points, while the French writers lay most stress on the applications of this subject in the theory of functions. M.

Alfred Ackermann-Teubner has given twenty thousand marks—about five thousand dollars—to the University of Leipzig, to establish a *mathematical prize.* The first award is to be made in 1914, and every two years thereafter until the surplus accumulations amount to sixty thousand marks. After this time the prize is to be awarded annually. The subjects for which the prize after 1914 is to be awarded are, in order, as follows:

1. History, philosophy, and teaching;
2. Mathematics, especially arithmetic and algebra;
3. Mechanics;
4. Mathematical physics;
5. Mathematics, especially analysis;
6. Astronomy and theory of errors;
7. Mathematics, especially geometry;
8. Applied mathematics, especially geodesy and geophysics.

The range of the subject matter is to be about that given in the large German mathematical encyclopedia, which is now being published by B. G. Teubner of Leipzig, Germany. M.

In the "Summary Report" on the teaching of mathematics in Japan, which was recently published, there is given, page 197, a list of third year courses in mathematics in the Tokio Imperial University. This is of interest as it indicates how advanced their higher courses in mathematics really are. Four courses, bearing the following general headings,—General Theory of functions, Theory of differential equations, Theory of numbers and algebra, Higher geometry,—are outlined as follows: Riemann's surface and